

Sum-of-Sinusoids-Based Simulation of Flat Fading Wireless Propagation Channels Under Non-Isotropic Scattering Conditions

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Abstract—This paper presents a modified version of the method of equal areas (MEA) for designing simulation models for frequency non-selective mobile fading channels under non-isotropic scattering conditions. The proposed method, called modified MEA (MMEA), is well suited for channel simulators based on a finite sum of complex sinusoids. The combination of the MMEA with the principle of set partitioning is also proposed here as an efficient way to improve the performance of the simulator and to reduce the computational costs. Such a combination results in a new parameter computation method called MMEA with set partitioning (MMEA-SP). The MMEA and the MMEA-SP are quite general and can be applied on any given distribution of the angle of arrival (AOA). However, to exemplify the good performance of both methods, it is assumed that the AOA follows the von Mises distribution. The obtained results demonstrate that the two proposed methods approximate the autocorrelation function (ACF) of non-isotropic scattering channels with high precision.

Keywords—Channel simulators, deterministic channel modeling, mobile fading channels, non-isotropic scattering, set partitioning, sum-of-sinusoids principle.

I. INTRODUCTION

The sum-of-sinusoids (SOS) principle introduced by Rice [1], [2] for the modeling of colored Gaussian processes has found widespread acceptance as an adequate basis for the design of simulation models for mobile fading channels [3]–[6]. SOS-based simulators can generate temporally correlated sequences with reasonably low computational costs. Owing to this characteristic, this kind of simulation models turn out to be especially suitable for simulating frequency non-selective wireless propagation channels with specified autocorrelation functions (ACFs). Furthermore, the SOS principle has been used to simulate efficiently frequency selective [7], [8] and spatial selective wireless channels [9]–[11].

The design of accurate and efficient SOS-based simulators for isotropic scattering channels has been the topic of research of several papers [3]–[9]. However, despite its importance, the SOS-based simulation of wireless channels under the more realistic scenario of non-isotropic scattering has not received much attention so far. To close this gap, we propose in this paper a modified version of the method of equal areas (MEA)

[3] which allows the design of SOS-based simulation models for mobile fading channels under non-isotropic scattering conditions. We also explain how to combine the proposed method, called modified MEA (MMEA), with the principle of set partitioning to reduce the computational costs associated with the generation of high-quality channel waveforms. The parameter computation method that results from such a combination is called MMEA with set partitioning (MMEA-SP). The idea of applying set partitioning to the simulation of mobile fading channels was originally proposed in [12] for the particular case of isotropic scattering channels. In this contribution, we extend the idea with respect to all kinds of non-isotropic scattering scenarios. We stress that the MMEA and the MMEA-SP can be applied on any given distribution of the angle of arrival (AOA). Nevertheless, to demonstrate the good performance of these two methods, we present some exemplary numerical results by assuming that the AOA is von Mises distributed [13].

In a related paper [14], we explain how to compute the simulation model's parameters of non-isotropic scattering channels with given asymmetrical Doppler power spectra by using the MMEA and the MMEA-SP. The differences between the approach described here and that in [14] will be highlighted.

The rest of the paper is organized as follows. In Section II, we give a brief description of the reference model and the SOS-based simulation model. In Section III, we present the MMEA. In Section IV, we explain how to combine the MMEA with the principle of set partitioning. In Section V, we discuss the differences between the MMEA-SP and other similar parameter computation methods, such as the one described in [14]. Finally, we present our conclusions in Section VI.

II. REFERENCE MODEL AND SIMULATION MODEL

A. The Reference Model

Our reference model is a frequency non-selective Gaussian channel, which characterizes a two-dimensional propagation environment, where the scattering is not necessarily isotropic. Invoking the central limit theorem [15], we can express such a channel model as an infinite sum of complex

harmonic functions in the form [12], [13]

$$\mu(t) = \lim_{N \rightarrow \infty} \frac{\sigma}{\sqrt{N}} \sum_{n=1}^N \exp\{j(2\pi f_{\max} \cos(\alpha_n)t + \theta_n)\} \quad (1)$$

where each complex harmonic function represents a propagation path, f_{\max} is the maximum Doppler frequency, α_n is a random variable denoting the AOA of the n th path, and the phases θ_n are independent identically distributed random variables, each having a uniform distribution over $[-\pi, \pi)$. An isotropic scattering channel results from (1) if the AOAs α_n are uniformly distributed over the interval $[-\pi, \pi)$. On the other hand, $\mu(t)$ describes a non-isotropic scattering channel if the AOAs α_n exhibit a nonuniform distribution.

B. The Simulation Model

We can observe from (1) that a hardware and/or software realization of the complex Gaussian process $\mu(t)$ is not possible, since $\mu(t)$ comprises an infinite sum of complex harmonic functions. Fortunately, we can satisfactorily approximate the reference model, characterized by $\mu(t)$, by a stochastic process

$$\hat{\mu}(t) = \frac{\sigma}{\sqrt{N}} \sum_{n=1}^N \exp\{j(2\pi f_{\max} \cos(\tilde{\alpha}_n)t + \theta_n)\} \quad (2)$$

where the number of harmonic functions is limited, usually $N \approx 20$, and the AOAs $\tilde{\alpha}_n$ are realizations of the random variables α_n . The statistical properties of $\hat{\mu}(t)$, such as the probability density function (PDF) and level-crossing rate, are analyzed in [16]. Here, we only need to know that the density of $\hat{\zeta}(t) = |\hat{\mu}(t)|$ is close to the Rayleigh distribution if $N \geq 20$. Furthermore, a deterministic simulator results from the stochastic process $\hat{\mu}(t)$ if we take single outcomes $\tilde{\theta}_n$ of the random phases θ_n . By doing so, we obtain a deterministic process (sample function) given by

$$\tilde{\mu}(t) = \frac{\sigma}{\sqrt{N}} \sum_{n=1}^N \exp\{j(2\pi f_n t + \tilde{\theta}_n)\} \quad (3)$$

which acts as simulation model in this paper. The Doppler frequencies f_n are defined as

$$f_n := f_{\max} \cos(\tilde{\alpha}_n), \quad n = 1, \dots, N. \quad (4)$$

The ACF of $\tilde{\mu}(t)$ is given by

$$\tilde{r}_{\mu\mu}(\tau) := \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T [\tilde{\mu}(t)]^* \tilde{\mu}(t + \tau) dt \quad (5)$$

$$= \frac{\sigma^2}{N} \sum_{n=1}^N \exp\{j2\pi f_n \tau\}. \quad (6)$$

Since we are dealing with complex Gaussian processes, the performance evaluation of the deterministic SOS-based simulation model reduces to the investigation of the quality of the approximation $\tilde{r}_{\mu\mu}(\tau) \approx r_{\mu\mu}(\tau)$ within $[0, \tau_{\max}]$, where $r_{\mu\mu}(\tau) := E\{\mu^*(t)\mu(t+\tau)\}$ is the ACF of the reference model $\mu(t)$ and τ_{\max} defines the length of the interval over which the approximation is of interest. The notation $E\{\cdot\}$ stands for the statistical expectation, while $\{\cdot\}^*$ denotes complex

conjugation. The simulation parameters $\tilde{\alpha}_n$, or equivalently f_n , must therefore be computed in such a way that the ACF $\tilde{r}_{\mu\mu}(\tau)$ of the simulation model fits well to the ACF $r_{\mu\mu}(\tau)$ of the reference model.

III. THE PARAMETER COMPUTATION METHOD

A. The MMEA

The problem consists in finding the set of AOAs $\{\tilde{\alpha}_n\}_{n=1}^N$ of the simulation model given by $\tilde{\mu}(t)$ in (3), which provides a reasonably good approximation $\tilde{r}_{\mu\mu}(\tau) \approx r_{\mu\mu}(\tau)$, $\tau \in [0, \tau_{\max}]$, for a given value of N ($N \geq 20$). We will not pay attention to the phases $\tilde{\theta}_n$, since the ACF $\tilde{r}_{\mu\mu}(\tau)$ of the simulation model does not depend on them [see (6)].

To cope with the parameter computation problem described above, we recall that the ACF $r_{\mu\mu}(\tau)$ of the reference model $\mu(t)$ depends on the PDF $p_\alpha(\alpha)$ of the AOA α of the received signal [17]. Taking this fact into account, we use the following criterion to compute the AOAs $\tilde{\alpha}_n$ of the simulation model:

$$\int_{\tilde{\alpha}_{n-1}}^{\tilde{\alpha}_n} p_\alpha(\alpha) d\alpha = \frac{1}{N}, \quad \tilde{\alpha}_n \in [-\pi, \pi) \quad (7)$$

for $n = 1, \dots, N$, where $\tilde{\alpha}_0 = -\pi$. To exhaust the potential that the simulation model $\tilde{\mu}(t)$ offers, it is necessary to meet the following conditions:

$$(i) \quad f_n \neq f_m, \quad m \neq n \quad (8)$$

$$(ii) \quad f_n \neq 0, \quad \forall n. \quad (9)$$

Condition (i) guarantees N effective harmonic functions for the simulation model $\tilde{\mu}(t)$, while Condition (ii) assures that the mean value of $\tilde{\mu}(t)$ equals zero. Given that $f_n = f_{\max} \cos(\tilde{\alpha}_n)$, it follows that

$$\text{to meet (i):} \quad \tilde{\alpha}_n \neq -\tilde{\alpha}_m, \quad \forall n, m \quad (10)$$

$$\text{to meet (ii):} \quad \tilde{\alpha}_n \neq \pm\pi/2, \quad \forall n. \quad (11)$$

Condition (i) is not satisfied if the PDF $p_\alpha(\alpha)$ is symmetric with respect to the origin and N is an even number. Furthermore, Condition (ii) does not hold if $p_\alpha(\alpha)$ is uniform over $[-\pi, \pi)$. Therefore, to guarantee the fulfillment of the inequalities in (10) and (11) under such a circumstances, we impose the initial condition

$$\int_{\tilde{\alpha}_0}^{\tilde{\alpha}_1} p_\alpha(\alpha) d\alpha = \frac{1}{N} - \frac{1}{4N}. \quad (12)$$

Consider now the cumulative distribution function $F_\alpha(\alpha)$ of the AOA α , which is defined as

$$F_\alpha(x) := \int_{-\infty}^x p_\alpha(\alpha) d\alpha. \quad (13)$$

Then, using (7) and (12) to evaluate $F_\alpha(\tilde{\alpha}_n)$, we obtain

$$\begin{aligned} F_\alpha(\tilde{\alpha}_n) &= \sum_{m=1}^n \int_{\tilde{\alpha}_{m-1}}^{\tilde{\alpha}_m} p_\alpha(\alpha) d\alpha \\ &= \frac{1}{N} \left(n - \frac{1}{4} \right). \end{aligned} \quad (14)$$

Hence, the parameters $\tilde{\alpha}_n$ can be obtained by solving

$$\int_{-\pi}^{\tilde{\alpha}_n} p_\alpha(\alpha) d\alpha = \frac{1}{N} \left(n - \frac{1}{4} \right), \quad n = 1, \dots, N \quad (16)$$

by using numerical root-finding techniques. If the inverse function F_α^{-1} of F_α exists, then we can compute the AOAs $\tilde{\alpha}_n$ in closed form in accordance to

$$\tilde{\alpha}_n = F_\alpha^{-1} \left[\frac{1}{N} \left(n - \frac{1}{4} \right) \right], \quad n = 1, \dots, N. \quad (17)$$

We call the parameter computation method described in this section the MMEA, since it has been inspired by the MEA proposed in [3]. It is important to mention, however, that the original MEA and the MMEA have been conceived for the simulation of different propagation scenarios, and therefore have dissimilar characteristics. The difference between the MEA and the MMEA will be explained in more detail in Section V.

B. Numerical Results for the von Mises Distribution

The von Mises distribution has been shown to be an adequate model for the AOA statistics of both isotropic and non-isotropic scattering channels [13]. This PDF is given by

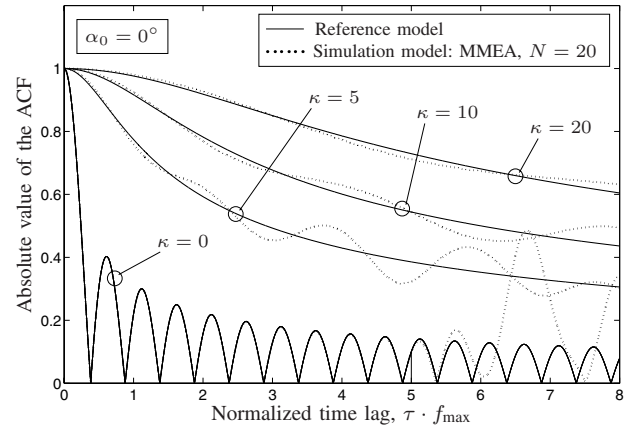
$$p_\alpha(\alpha) = \frac{\exp\{\kappa \cos(\alpha - \alpha_0)\}}{2\pi I_0(\kappa)}, \quad \alpha \in [-\pi, \pi) \quad (18)$$

where $\alpha_0 \in [-\pi, \pi)$ is the mean AOA, $\kappa \geq 0$ is a parameter that determines the angular spread, and the symbol $I_0(\cdot)$ stands for the zeroth order modified Bessel function. By adopting the von Mises PDF for the reference model $\mu(t)$, then its ACF $r_{\mu\mu}(\tau)$ can be expressed as [13]

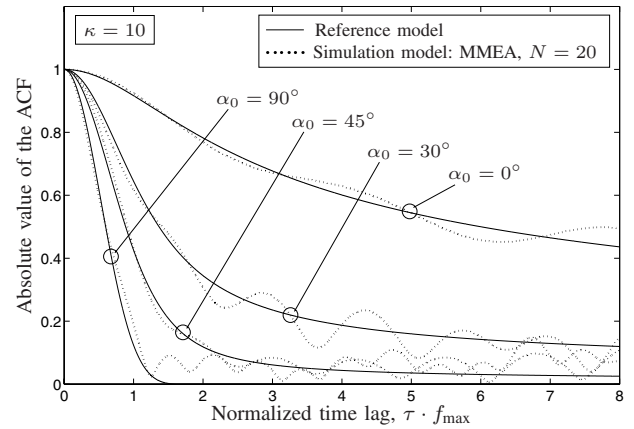
$$r_{\mu\mu}(\tau) = \frac{I_0 \left(\sqrt{\kappa^2 - (2\pi f_{\max} \tau)^2 + j4\pi\kappa f_{\max} \cos(\alpha_0) \tau} \right)}{I_0(\kappa)}. \quad (19)$$

If $\kappa = 0$, then the ACF $r_{\mu\mu}(\tau)$ reduces to the well-known ACF $r_{\mu\mu}(\tau) = J_0(2\pi f_{\max} \tau)$ characterizing isotropic scattering channels [17], where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind.

Figure 1 shows a comparison between the ACF $r_{\mu\mu}(\tau)$ given in (19) for $\mu(t)$ and the ACF $\tilde{r}_{\mu\mu}(\tau)$ of the simulation model $\tilde{\mu}(t)$ designed by using the MMEA with $N = 20$ for various values of α_0 and κ . We computed the AOAs $\tilde{\alpha}_n$ by using numerical methods to solve (16). From Fig. 1, we can conclude that the MMEA provides a good approximation of the ACF $r_{\mu\mu}(\tau)$ of the reference model. The MMEA has an excellent performance indeed when the scattering is isotropic ($\kappa = 0$) and $\tau \in [0, N/(4f_{\max})]$. It is interesting to note that in case of isotropic scattering ($\kappa = 0$), the MMEA reduces to the method of exact Doppler spread (MEDS) [4]. Also, it is worth to mention that the quality of the approximation $r_{\mu\mu}(\tau) \approx \tilde{r}_{\mu\mu}(\tau)$ can significantly be improved for any $\kappa \geq 0$ by increasing the number of complex harmonic functions N .



(a) $\alpha_0 = 0, \kappa = \{0, 5, 10, 20\}$



(b) $\kappa = 10, \alpha_0 = \{0^\circ, 30^\circ, 45^\circ, 90^\circ\}$

Fig. 1. Comparison between the absolute value of the ACF $|r_{\mu\mu}(\tau)|$ of the reference model and the absolute value of the ACF $|\tilde{r}_{\mu\mu}(\tau)|$ of the simulation model by using the MMEA with $N = 20$ for various values of α_0 and κ .

IV. EFFICIENT SIMULATION OF HIGH-QUALITY CHANNEL WAVEFORMS

It was recently shown in [12] that a simulation approach based on set partitioning allows to improve the performance of deterministic SOS-based simulation models simply by averaging across several simulation runs. This approach is quite advantageous, since the performance of deterministic SOS-based simulators could only be improved until then by increasing the number of sinusoids. The method presented in [12] was originally proposed for the simulation of isotropic scattering environments. In this section, we generalize the procedure with respect to all kinds of non-isotropic scattering scenarios.

A. The MMEA-SP

Following the approach proposed in [12], we have to generate K uncorrelated channel waveforms

$$\tilde{\mu}^{(k)}(t) = \frac{\sigma}{\sqrt{N}} \sum_{n=1}^N \exp\{j(2\pi f_n^{(k)} t + \tilde{\theta}_n^{(k)})\} \quad (20)$$

with $f_n^{(k)} = f_{\max} \cos(\tilde{\alpha}_n^{(k)})$, such that the sample mean ACF

$$\bar{r}_{\mu\mu}(\tau) := \frac{1}{K} \sum_{k=1}^K \tilde{r}_{\mu\mu}^{(k)}(\tau) \quad (21)$$

equals the ACF $\tilde{r}_{\mu\mu}(\tau)$ of a single waveform $\tilde{\mu}(t)$ composed by NK complex harmonic functions. In (21), $\tilde{r}_{\mu\mu}^{(k)}(\tau)$ is the ACF of the k th waveform $\tilde{\mu}^{(k)}(t)$. In other words, we have to generate K complex waveforms $\tilde{\mu}^{(k)}(t)$ such that

$$\bar{r}_{\mu\mu}(\tau) = \tilde{r}_{\mu\mu}(\tau), \quad -\infty \leq \tau \leq \infty \quad (22)$$

$$\tilde{r}_{\mu\mu}^{(k,l)}(\tau) = 0, \quad k \neq l \quad (23)$$

where

$$\tilde{r}_{\mu\mu}^{(k,l)}(\tau) := \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T [\tilde{\mu}^{(k)}(t)]^* \tilde{\mu}^{(l)}(t + \tau) dt \quad (24)$$

is the CCF of $\tilde{\mu}^{(k)}(t)$ and $\tilde{\mu}^{(l)}(t)$. Note that the ACF $\tilde{r}_{\mu\mu}^{(k)}(\tau)$ of $\tilde{\mu}^{(k)}(t)$ results from (24) if $k = l$.

By assuming that $f_n^{(k)} \neq f_m^{(k)}$ for all k and $n \neq m$, the parameter computation problem at hand reduces to finding K sets $\{\tilde{\alpha}_n^{(k)}\}_{n=1}^N$ that satisfy the following conditions:

$$(iii) \quad \{\tilde{\alpha}_n^{(k)}\}_{n=1}^N \cap \{\tilde{\alpha}_m^{(l)}\}_{m=1}^N = \emptyset, \quad k \neq l \quad (25)$$

$$(iv) \quad \bigcup_{k=1}^K \{\tilde{\alpha}_n^{(k)}\}_{n=1}^N = \{\tilde{\alpha}_n\}_{n=1}^{M=NK} \quad (26)$$

where \emptyset is the empty set, and $\{\tilde{\alpha}_n\}_{n=1}^{M=NK}$ is the set composed by the AOs $\tilde{\alpha}_n$ of a waveform $\tilde{\mu}(t)$ obtained by using the MMEA with $M = NK$ complex harmonic functions.

The conditions stated in (25) and (26) are fulfilled if the AOs $\tilde{\alpha}_n^{(k)}$ of the K channel waveforms $\tilde{\mu}^{(k)}(t)$ are computed in accordance to

$$\tilde{\alpha}_n^{(k)} = \tilde{\alpha}_{k+(n-1)K}, \quad n = 1, \dots, N, \quad k = 1, \dots, K. \quad (27)$$

In this manner, we also assure that the area under the PDF $p_\alpha(\alpha)$ equals $1/N$ within $[\tilde{\alpha}_n^{(k)}, \tilde{\alpha}_{n+1}^{(k)})$ for $n = 1, \dots, N - 1$. From (27) and (16), we have

$$\begin{aligned} F_\alpha(\tilde{\alpha}_n^{(k)}) &= \int_{-\pi}^{\tilde{\alpha}_{k+(n-1)K}} p_\alpha(\alpha) d\alpha \\ &= \frac{1}{KN} \left(k + (n-1)K - \frac{1}{4} \right) \end{aligned} \quad (28)$$

for $n = 1, \dots, N$, and $k = 1, \dots, K$. The previous result can be rewritten in the form

$$\int_{-\pi}^{\tilde{\alpha}_n^{(k)}} p_\alpha(\alpha) d\alpha = \frac{1}{N} \left(n - \frac{1}{4} \right) + \epsilon_k \quad (29)$$

where

$$\epsilon_k = \frac{4k - 3K - 1}{4KN} \quad (30)$$

is called the angle of rotation. Then, we can compute the AOs $\tilde{\alpha}_n^{(k)}$ of the k th waveform $\tilde{\mu}^{(k)}(t)$ by solving (29) with the aid of numerical root-finding algorithms. This parameter computation method establishes the so-called MMEA-SP. Notice that the MMEA-SP reduces to the MMEA if $K = 1$, i.e., $\epsilon_k = 0$.

In the special case where the inverse F_α^{-1} of F_α exists, the AOs $\tilde{\alpha}_n^{(k)}$ can be obtained from the closed-form solution

$$\tilde{\alpha}_n^{(k)} = F_\alpha^{-1} \left(\frac{1}{N} \left[n - \frac{1}{4} \right] + \epsilon_k \right). \quad (31)$$

B. Numerical Results for the von Mises Distribution

Figure 2 shows a comparison between the sample mean ACF $\bar{r}_{\mu\mu}(\tau)$ obtained by using the MMEA-SP with $N = 20$ and $K = 4$, and the ACF $r_{\mu\mu}(\tau)$ given in (19) for the reference model. We considered for the simulations the same values of α_0 and κ as in Section III.B. It is evident that the results presented in Figs. 2(a) and 2(b) outperform by far those in Figs. 1(a) and 1(b), respectively. Indeed, the sample mean ACF $\bar{r}_{\mu\mu}(\tau)$ show less deviations from $r_{\mu\mu}(\tau)$ than the ACF $\tilde{r}_{\mu\mu}(\tau)$ of a single waveform obtained by using the MMEA with $N = 20$ [cf. Fig. 1]. Interestingly, a quick inspection of the isotropic scattering case ($\kappa = 0$) reveals that the MMEA-SP yields an excellent approximation to the ACF $r_{\mu\mu}(\tau)$ of the reference model for an interval K times larger than in the corresponding example presented in Fig. 1(a) for the MMEA.

V. DIFFERENCES BETWEEN THE MMEA-SP AND SIMILAR PARAMETER COMPUTATION METHODS

In a related paper [14], we proposed the MMEA-SP as an efficient method to compute the Doppler frequencies $f_n^{(k)}$ of the k th waveform $\tilde{\mu}^{(k)}(t)$ by solving

$$\int_{-f_{\max}}^{f_n^{(k)}} S_{\mu\mu}(f) df = \frac{1}{N} \left(n - \frac{1}{2} \right) + \xi_k, \quad n = 1, \dots, N \quad (32)$$

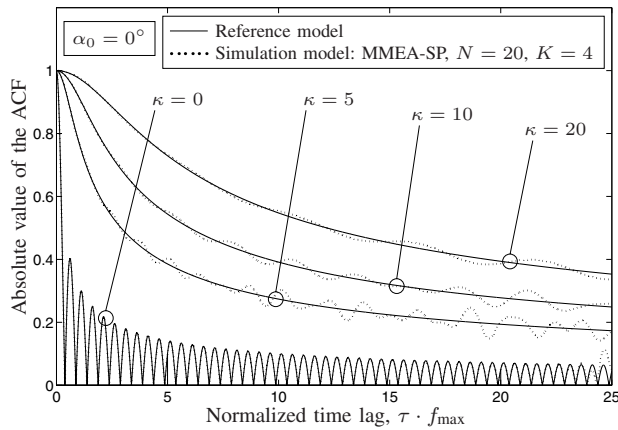
where $S_{\mu\mu}(f)$ is a given bandlimited (normalized) Doppler power spectrum of the reference model $\mu(t)$, and ξ_k is defined as

$$\xi_k = \frac{1}{KN} \left(k - \frac{K+1}{2} \right), \quad k = 1, \dots, K. \quad (33)$$

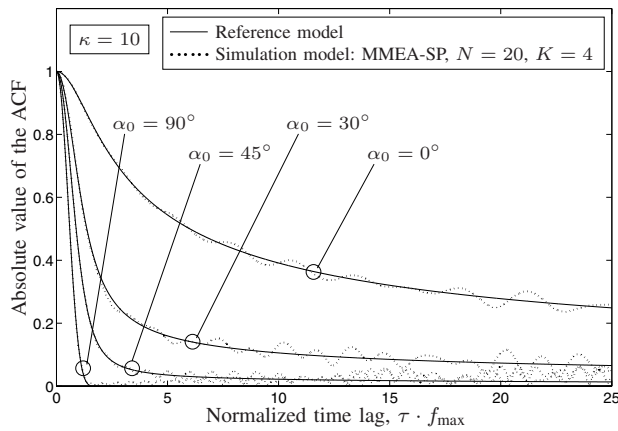
Since the Doppler frequencies $f_n^{(k)}$ are related to the AOs $\tilde{\alpha}_n^{(k)}$ via $f_n^{(k)} = f_{\max} \cos(\tilde{\alpha}_n^{(k)})$, one can show that the AOs $\tilde{\alpha}_n^{(k)}$ computed from (29) result in the same values for $f_n^{(k)}$ as the solution of (32). However, the Doppler frequencies $f_n^{(k)}$ computed by using (32) do not produce the same AOs as those obtained from (29). This is because the AOs $\tilde{\alpha}_n^{(k)} = \arccos(f_n^{(k)}/f_{\max})$ are confined on the interval $[0, \pi)$, while the AOs obtained from (29) are located in the whole range $[-\pi, \pi)$. This difference explains the discrepancy between the right-hand-side terms of the expression in (32) and (29).

If the PDF $p_\alpha(\alpha)$ is symmetric with respect to the origin, then we can compute the AOs $\tilde{\alpha}_n^{(k)}$ from $p_\alpha(\alpha)$ by considering only the range $[0, \pi)$, because the rest of the interval is redundant (since the interval $[-\pi, 0)$ provides us with the same information as $[0, \pi)$). For this particular case, the integral in (29) can be redefined to produce the same result as the integral in (32).

Actually, if the PDF $p_\alpha(\alpha)$ is uniformly distributed over $[-\pi, \pi)$, which means that the scattering is isotropic, then the AOs of the simulation model can accurately be computed



(a) $\alpha_0 = 0, \kappa = \{0, 5, 10, 20\}$



(b) $\kappa = 10, \alpha_0 = \{0^\circ, 30^\circ, 45^\circ, 90^\circ\}$

Fig. 2. Comparison between the absolute value of the ACF $|r_{\mu\mu}(\tau)|$ of the reference model and the absolute value of the sample mean ACF $|\bar{r}_{\mu\mu}(\tau)|$ of the simulation model by using the MMEA-SP with $N = 20$ and $K = 4$ for various values of α_0 and κ .

by considering a shorter range $[0, \pi/2)$. This characteristic is exploited by the original MEA [3] and by the method proposed in [12] to make an efficient use of the number of harmonic functions. Unfortunately, such an interval is too short in general for the adequate characterization of non-isotropic scattering scenarios.

Finally, if the PDF $p_\alpha(\alpha)$ is asymmetric with respect to the origin, then it is necessary to consider the complete range $[-\pi, \pi)$. This is especially important if the distribution of the AOAs of the received signal is concentrated around a mean AOA α_0 that is different from zero, i.e., $\alpha_0 \neq 0$. Since the MMEA-SP (as well as the MMEA) has been designed to cover the relevant interval $[-\pi, \pi)$, we can surely claim that this method is valid for any given distribution of the AOA.

VI. CONCLUSIONS

In this paper, we presented a modified version of the MEA for the design of simulation models for non-isotropic scattering channels. Such a method, which we have called the MMEA, is quite general and can be applied on any

given distribution of the AOA. From the results presented in the paper, we can conclude that the MMEA is an adequate parameter computation method for the SOS-based simulation of non-isotropic scattering channels. In addition, we explained how to combine the MMEA with the principle of set partitioning to improve the simulator's performance and keep the computational costs low. Such a combination results in the parameter computation method called MMEA-SP. We have demonstrated the effectiveness and accuracy of the MMEA-SP by means of some numerical examples assuming that the AOA of the received signal follows the von Mises distribution.

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